TEMPERATURE STRATIFICATION IN CLOSED LIQUID-FILLED VOLUMES WITH CONSTANT HEAT-FLUX DENSITY AT THE SHELL

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A one-dimensional model is proposed for the estimation of temperature stratification in closed liquid-filled volumes. The results obtained are compared with experimental data and the results of numerical solution.

When low-boiling (in particular, cryogenic) liquids are stored in closed volumes, temperature stratification is observed, in that the liquid temperature T_s close to its boundary with the gas is considerably higher than the mean-mass temperature $\bar{T}$. The temperature stratification is accompanied by a significant pressure increase in the volume – much larger than in uniform liquid heating. Both these effects (intense increase in the surface temperature of the liquid and the pressure with time) are extremely undesirable, since they limit the storage time and create considerable difficulties for the use of liquids [1–3].

The process occurring in a heat-insulated volume containing liquid at a temperature below the temperature of the surrounding medium may be regarded as internal convection with boundary conditions of the second kind (a given constant heat-flow density at the shell). Analytic solution of such problems is associated with considerable and often insurmountable difficulties [4]. Numerical solutions may only be found for certain shell configuration in a limited range of Rayleigh and Fourier numbers [5]. Essentially, the various empirical generalizations of the experimental data (for example, [2, 6, 7]) are only true for the materials and experimental conditions for which they are obtained. Thus, there is at present no reliable method (not only of calculation but of estimating the temperature stratification).

Below a simple method is proposed for the estimation of the temperature stratification.

Consider the heating of a liquid partially filling a volume of arbitrary shape (for example, spherical; see Fig. 1). Suppose that at time $\tau = 0$ the shell containing the liquid at temperature T_0 begins to be heated by a heat flux of constant density q. To determine the temperature field in the liquid at subsequent times the following assumptions are made:

1) that the temperature field in the liquid is one-dimensional;

2) that the heat flux $Q_l = qF_l$ incident on the part of the shell bounding the liquid is consumed in uniform heating of the liquid, i.e., in increasing its mean-mass temperature;

3) that the heat flux $Q_g = gF_g$ incident on the part of the shell bounding the gas (vapor) is consumed in heating the liquid–gas boundary. The heat in the liquid propagates as in a semibounded body but with effective thermal conductivity $\lambda_{\text{eff}} = \varepsilon\lambda$, where $\varepsilon = f(\text{Ra})$;

4) that the effects of the temperature fields due to the heat fluxes Q and Q_g are additive (it has been experimentally confirmed that the temperature field in the heating of the shell surfaces F_l and F_g is practically additive [3, 7]);

5) that the heat flux consumed in vaporizing the liquid and heating the shell and the gas (vapor) is negligible.

Under these assumptions, the temperature field in the liquid may be written as

$$\Theta\left(\frac{x}{R}, \text{Fo}\right) = \bar{\Theta} + \Theta_\lambda\left(\frac{x}{R}, \text{Fo}\right). \tag{1}$$

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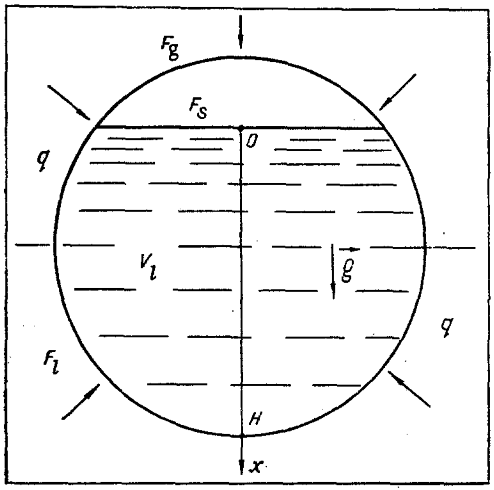

Fig. 1. Partially filled shell.

The dimensionless mean-mass temperature is easily found from the heat balance:

$$qF_l\,\tau = V_l\,\rho c\,(\bar{T} - T_0). \tag{2}$$

Rearranging Eq. (2), the following result is obtained:

$$\bar{\Theta} = \frac{(\bar{T} - T_0)\lambda}{qR} = \frac{F_l\,R}{V_l}\,\mathrm{Fo}. \tag{3}$$

For example, in the case of a spherical shell

$$\frac{F_l\,R}{V_l} = \frac{3H}{2mR}, \tag{4}$$

where

$$m = \frac{H^2}{4R^2}\left(3 - \frac{H}{2R}\right) = \frac{V_l}{V}$$

is a coefficient characterizing the filling of the volume by the liquid.

The expression for the temperature due to heat flux Q_g may be found (under the given assumptions) from the solution for a semibounded body [8] in the form

$$\Theta_\lambda\left(\frac{x}{R},\,\mathrm{Fo}\right) = \frac{2F_g}{F_s\sqrt{\varepsilon}}\sqrt{\mathrm{Fo}}\;\mathrm{ierfc}\,\frac{x}{R}\,\frac{1}{2\sqrt{\varepsilon}\sqrt{\mathrm{Fo}}}. \tag{5}$$

In the case of a spherical shell $F_g/F_s = 2R/H$.

In the first approximation, the dependence of $\varepsilon = \lambda_{\mathrm{eff}}/\lambda$ on the Rayleigh number may be taken to be the same as in the calculation of the effective thermal conductivity in layers [9]

$$\varepsilon = A\,\mathrm{Ra}^{1/6}. \tag{6}$$

In the present case the coefficient A, which in the calculation of the heat transfer is constant, may depend on the shell configuration, the filling coefficient m, the fraction of heat consumed in liquid vaporization, and several other factors.

Finally, substituting Eq. (6) into Eq. (5) and Eqs. (3) and (5) into Eq. (1) gives

$$\Theta\left(\frac{x}{R},\,\mathrm{Fo}\right) = \frac{F_l\,R}{V_l}\,\mathrm{Fo} + \frac{2F_g\sqrt{\mathrm{Fo}}}{\sqrt{A}\,F_s\,\mathrm{Ra}^{1/12}}\;\mathrm{ierfc}\,\frac{x}{2\sqrt{A\,\mathrm{Fo}}\,R\,\mathrm{Ra}^{1/12}}. \tag{7}$$

Noting that ierfc 0 = 0.564 [8], the liquid-surface temperature $\Theta_s = \Theta(0,\ \mathrm{Fo})$ may be written in the form

$$\Theta_s = \frac{F_l\,R}{V_l}\,\mathrm{Fo} + \frac{1.13F_g\sqrt{\mathrm{Fo}}}{\sqrt{A}\,F_s\,\mathrm{Ra}^{1/12}}. \tag{8}$$

It is of interest to compare the results obtained using Eq. (8) with those obtained experimentally and in numerical calculations. The liquid-surface temperature Θ_s is determined using the approximate values $\Theta = 3\mathrm{Fo}$, $F_g \approx F_s$, and the experimental value [2] $\sqrt{A} \approx 0.2$. Then Eq. (8) may be replaced by the simplified expression

$$\Theta_s \approx 3\text{Fo} + 5.64\,\frac{\sqrt{\text{Fo}}}{\text{Ra}^{1/12}}. \tag{9}$$

Curves of $\Theta_S = \Theta_S(\text{Fo})$ calculated from Eq. (9) for $\text{Ra} = 10^{10}$, 10^{11}, and 10^{12} are shown in Fig. 2, together with experimental data for spherical shells partially filled with nitrogen [2] and water [7] and averaged data for water and alcohols at complete filling of the volume [6]. As is evident from Fig. 2, the discrepancy between the experimental and calculated results is no more than ±15% for $\text{Fo} = 4 \cdot 10^{-3}$–0.2, despite the substantial simplification of the original model.

For uniform three-dimensional heating of a liquid enclosed in a cylindrical shell of height H = 2R, the liquid-surface temperature may be described by the "accurate" (within the framework of the given model) expression

$$\Theta_s = 2.08\,\text{Fo} + \frac{1{,}13\sqrt{\text{Fo}}}{\sqrt{A}\,\text{Ra}^{1/12}}. \tag{10}$$

The results obtained from Eq. (10) for the same value of A as for a sphere (A = 0.04) are 25% higher, on average, then the corresponding numerical solution [10]. If A = 0.08 for a vertical cylinder, the results given by Eq. (10) agree with the numerical solution (see Fig. 3) for a Fourier number of 0.01–0.1 differing by ±15% from the boundaries of the Fourier-number range ($\text{Fo} = 2.5 \cdot 10^{-3}$, 0.3) in which the numerical solution was carried out.

Using the proposed model, the temperature profile of the stratification region [see Eq. (5)] can be calculated, as well as the depth of the stratification region, the time at which quasisteady conditions begin, and the maximum possible (for the given value of Ra) temperature drop in the liquid.

Thus, the depth of the stratification region for the given error of the stratification determination may be calculated from Eq. (5). Taking this error to be 3% of the maximum stratification-region temperature $\Theta_\lambda(0, \text{Fo})$ gives

$$\frac{\Theta_\lambda\left(\frac{x}{R},\ \text{Fo}\right)}{\Theta_\lambda(0,\ \text{Fo})} = \frac{\text{ierfc}\,\dfrac{x}{2\sqrt{A\text{Fo}}\,\text{Ra}^{1/12}R}}{0{,}564} = 0{,}03$$

or

$$\text{ierfc}\,\frac{x}{2\sqrt{A\text{Fo}}\;\text{Ra}^{1/12}R} = 0.0166. \tag{11}$$

If the function ierfc has the value given in Eq. (11) the argument must be equal to 1.33; therefore

$$x/R \approx 2{,}66\,\text{Ra}^{1/12}\sqrt{A\text{Fo}}. \tag{12}$$

It is of interest to note that the depth of the stratification region determined experimentally in [2] varies analogously with time ($x/R \sim \text{Fo}^{0.6}$).

What are the limits of application of the proposed model? It may be assumed that the model is at least valid in the range $\text{Ra} = 10^6$–10^{12}. The lower value of the Fourier number is determined by the time to establish circulatory motion in the liquid Fo_l, after which liquid heating from above predominantly by heat conduction begins [10, 11]. This time may be estimated using the formula proposed in [11]:

$$\text{Fo}_l = 0{,}32\,\text{Ra}^{-0{,}2}. \tag{13}$$

For $\text{Ra} \approx 10^6$–10^{12} the approximate range of the lower limit on the time is $\text{Fo}_l \approx 10^{-2}$–$10^{-3}$.

The upper limit of the use of the model over time Fo_W is determined by the situation when the stratification descends to the "bottom" of the volume. For $\text{Fo} \geq \text{Fo}_W$ quasisteady conditions may be expected to begin, i.e., the invariability over time of the temperature difference between any two points in the liquid. The heating of the whole liquid mass will be the same as for a plate with a given temperature profile for a constant heat-flux density at its surface [8]. Finally, this scheme is very conventional: The stratification cannot descend to the "bottom" since its lower part is washed away by hot liquid jets issuing from the bottom of the volume. In addition, when quasisteady conditions begin the heat fluxes at the free surface of the liquid and at the lower region of stratification should be the same. However, this scheme allows the order of magnitude of Fo_W to be calculated. In fact, Fo_W can be determined from Eq. (12), setting x = 2R (which is true for both a spherical volume and a cylindrical volume with H = 2R). Then Eq. (12) gives the result

$$\text{Fo}_W = \frac{0{,}56}{A\,\text{Ra}^{1/6}}. \tag{14}$$

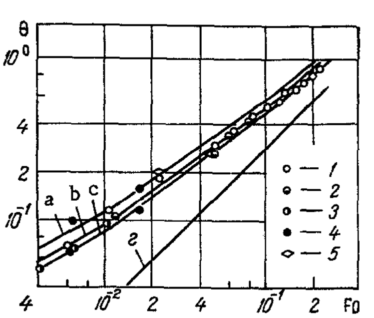

Fig. 2

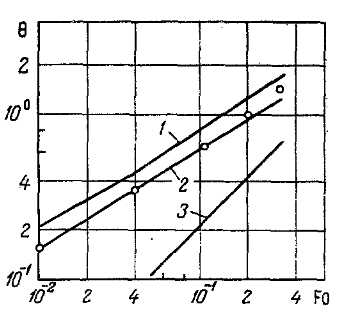

Fig. 3

Fig. 2. Comparison of calculated and experimental data (spherical volume). Calculated data: a) $Ra = 10^{10}$; b) 10^{11}; c) 10^{12}; d) $\bar{\Theta} = 3Fo$. Experimental data: 1) $Ra = 2.1 \cdot 10^{11}$; 2) $4.2 \cdot 10^{11}$; 3) $2.1 \cdot 10^{12}$, nitrogen, $m = 0.95$ [3]; 4) $Ra = 10^{10}$, 10^{11}, water and alcohols, $m = 1$ [6]; 5) $Ra = 10^{11}$, water $m = 0.85$ [7].

Fig. 3. Comparison of results obtained from Eq. (10) and by numerical solution [10] (cylindrical volume, $H = 2R$, $Ra = 10^6$, $m = 1$). The points are for $A = 0.04$; curve 1 is for $A = 0.08$; curve 2 is the numerical solution in [10]; curve 3 is for $\bar{\Theta} = 2.08\,Fo$, from Eq. (10).

For a cylinder containing liquid when $Ra = 10^6$, $H = 2R$, and $A = 0.08$, the result is $Fo_w = 0.7$; this agrees with the data of [10], where even for $Fo \approx 0.5$ no quasisteady conditions were observed.

In the case of a liquid in a spherical shell ($A = 0.04$) the range of the upper limit on the Fourier numbers for which the model may be used is $Fo_w \approx 1.4-0.14$, for Rayleigh numbers in the range 10^6-10^{12}.

Finally, substituting Eq. (14) into Eq. (8), the maximum value of the temperature stratification corresponding to quasisteady conditions is obtained; for large filling ($F_g \approx F_s$)

$$\Theta_{\lambda\max} = \frac{0{,}85}{A\,Ra^{1/6}}. \qquad (15)$$

In the case of a spherical shell with $Ra = 10^6-10^{12}$ the maximum temperature drop inside the liquid is found to be $\Theta_{\lambda\max} \approx 2-0.2$. For a cylindrical volume with $H/R = 2$ and $Ra = 10^6$, Eq. (15) gives $\Theta_{\lambda\max} = 1.06$; this is in satisfactory agreement with [10], where for near-quasisteady conditions ($Fo = 0.3$) the maximum vertical temperature drop in the volume is $\Theta_{\lambda\max} \approx 0.9$. The surface temperature Θ_s for $Fo \geqslant Fo_w$ may evidently be determined from Eq. (15).

This model of stratification not only gives a satisfactory quantitative description of the experimental results but also permits a simple explanation of several qualitative features of heat transfer in closed volumes for boundary conditions of the second kind. For example, the model gives a clear interpretation of the well-known increase in surface temperature and in temperature stratification with decrease in Ra (or decrease in the acceleration due to gravity) [5]. Admittedly, the model does not permit a transition in the limit to weightlessness and is true only in the range of sufficiently Rayleigh numbers (for example for $Ra > 10^5$ is $Fo \approx Fo_w$ [5]). Another example is the explanation of the different temperature variation with time at different points in the liquid (along the vertical axis). Thus in a certain range of Fourier numbers the model gives an approximate dependence $\Theta(x/R) \sim Fo^n$, where for the unstratified liquid core ($x \approx H$) $n = 1$ and for the surface ($x = 0$) $0.5 < n < 1$; for $Fo = 0.01-0.1$, $n \approx 0.6-0.8$, which is observed in experiments [3] and in the numerical solution [10] (see Figs. 2 and 3).

All the examples illustrating the use of this model refer to the case of large filling ($m \geq 0.85$). For significantly smaller values of m, the expression for the effective thermal conductivity λ_{eff} will evidently be more complex. In this case the specific form of the expression for λ_{eff} may be determined on the basis of an analysis of the corresponding experimental data and is an independent problem.

In conclusion, note that the model of temperature stratification outlined earlier [12], taking into account the shell configuration and the increase in mean-mass liquid temperature reduces to a one-dimensional differential heat-conduction equation with a convective term; the solution of this equation may only be obtained using numerical methods. Because of this, and also of the indeterminacy of the expression for the velocity appearing in the convective term, it is impossible to obtain reliable and to any degree general results. In the model which is considered in the present work, the effect of liquid motion and the shell shape on the heat propagation may be taken into account by an appropriate choice of the effective thermal conductivity.

NOTATION

A, coefficient in Eq. (6); a, thermal diffusivity; c, specific heat at constant pressure; F, surface area; g, acceleration due to gravity; H, height of liquid in volume; $m = V_l / V$, filling coefficient; Q, heat flux, q, heat-flux density; R, characteristic dimension (radius) of volume; T, temperature; $\bar{T}$, mean-mass temperature; $\Delta T = T - T_0$; T_0, initial temperature; V, volume; x, distance along volume axis, downward from liquid surface; β, volume-expansion coefficient; $\varepsilon = \lambda_{eff}/\lambda$; λ, λ_{eff}, thermal conductivity and effective thermal conductivity; ν, kinematic viscosity; ρ, density; τ, time; $\Theta = \Delta T\lambda/qR$, dimensionless temperature; $Fo = a\tau/R^2$, Fourier number; $Ra = gR^4\beta q/\nu a\lambda$, Rayleigh number. Indices: g, gas; l, liquid; s, gas–liquid interface.

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